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BAYESIAN APPROACH APPLIED TO AUTHENTICITY TESTING BY LUMINESCENCE

1. Introduction

In luminescence dating, the age is given by the ratio of the dose (calling the equivalent dose) from radioactive sources absorbed by the artefact since its last heat or exposure to light to the annual dose rate.

Assuming a constant dose rate on the time, it could be estimated from the present value of the ambient dose reported on one year. Natural irradiations composing the dose could be divided in four parts: the alpha particles component, the beta particles component, the gamma radiation and the cosmic rays. Due to their various length ranges, their origin are different. Alpha and beta have short range into minerals as quartz or feldspars: namely few microns for the alpha and less than two millimetres for beta. Hence, sampling the artefact more than two millimetres from the surface avoids the presence of such components from outside. On an other hand, the penetration range of the gamma reaches 30 cm, while the cosmic rays have quasi-infinite range. The variation on the cosmic dose is mainly due to the latitude and the depth from earth's surface and could be estimated using formula (Prescott, Hutton 1994). The gamma dose depends but of the surrounding soil. In the case of archaeological artefact, samples from the soil are measured in laboratory, or direct *in-situ* radiation measurements are made at the artefact's location. In the case of the museum objects, such a measurement is not possible due to the ignorance of the previous locations.

Hence for a museum object, where the main purpose is to distinct an original piece from a forgery, only the equivalent dose is used. A forgery shows very low emission when authentic object shows intense emission. In this case, "Authenticity testing" is used instead of dating.

Even if an age is obtained, it cannot be exploited without some remarks. Hence, it is not establish that the last heat for a ceramic corresponds to the heat during the manufacture process. Some subsequent fire, accidental or not, can have reset the signal and rejuvenate the object. As only few milligrams are sampling, the homogeneity of the artefact must be assumed. An other important warning concerns the possibility of some artificial irradiation.

Considering the various possibilities to lead to an overestimation or an underestimation, is the equivalent dose more in agreement with a genuine object or with a fake? For that, the two hypotheses: genuine or fake have to be compared.

2. Model

Our aim is then to test the equivalent dose with the two hypotheses. Hence, we must model our previous assumption on the equivalent dose. We have first to estimate the present annual dose. Although we don't know a precise value of the external dose, we can roughly estimate it based on some assumption and previous information. We need to evaluate too the probable age range. The product of the two distributions (age and dose rate) gives a first assumption of the equivalent dose. To be completed, the distribution of probability of artificial irradiations must be added.

3. Reconstitution of gamma

3.1 *Proportion of unknown gamma*

To estimate the part of unknown gamma, we need first to split the gamma between inner gamma, known from the radioelements content into the artefact and the external gamma, fully unknown. For such a purpose, we model the artefact by a semi-infinite plan, with a width equal to the mean size of the true artefact. Using the equation for the absorbed dose into an inert soil and the principle of superposition, we obtain the following system for the gamma irradiation (Aitken 1985, Appendix H):

$$D\gamma = D\gamma_{int}\,[f(x) + f(r) - 1] + D\gamma_{ext}\,[2 - f(x) - f(r)]$$

where $D\gamma_{int}$ is the inner gamma dose, $D\gamma_{ext}$ the external gamma dose, x the mean size of the artefact and r the distance from the closest surface of the sampling location, namely two millimetres. It is assumed that x is very great compared to r. f(x) is the fractional dose at the distance x from the surface, according to Løvborg's calculations cited by Aitken (1985, table H.1).

The uncertainty on the external gamma dose σext introduces an uncertainty on the age of $\sigma = 100\times(2\text{-}f(2mm)\text{-}f(r))$ x $\sigma ext / D$ where D is the total dose.

3.2 *Prior information on the gamma*

In the Nature, $D\gamma_{ext}$ cannot take all the values but a limited range. It could be estimated using a database of compiled gamma external doses reported in the literature, as well as our own measured values. More than 400 values coming from about 50 references are plotted on Fig. 1. No value is observed above 4 mGy/a, which give a maximum limit. Two peaks are identified at 0.3mGy/a and 1.0mGy/a, which could be associated to low radioactive soils as rich-quartz loess and to high radioactive soils as rich-feldspathic rocks (granite, lava). Such histogram could be simulated by different ways. Among others, we can suggest the combination of two Gaussian distribu-

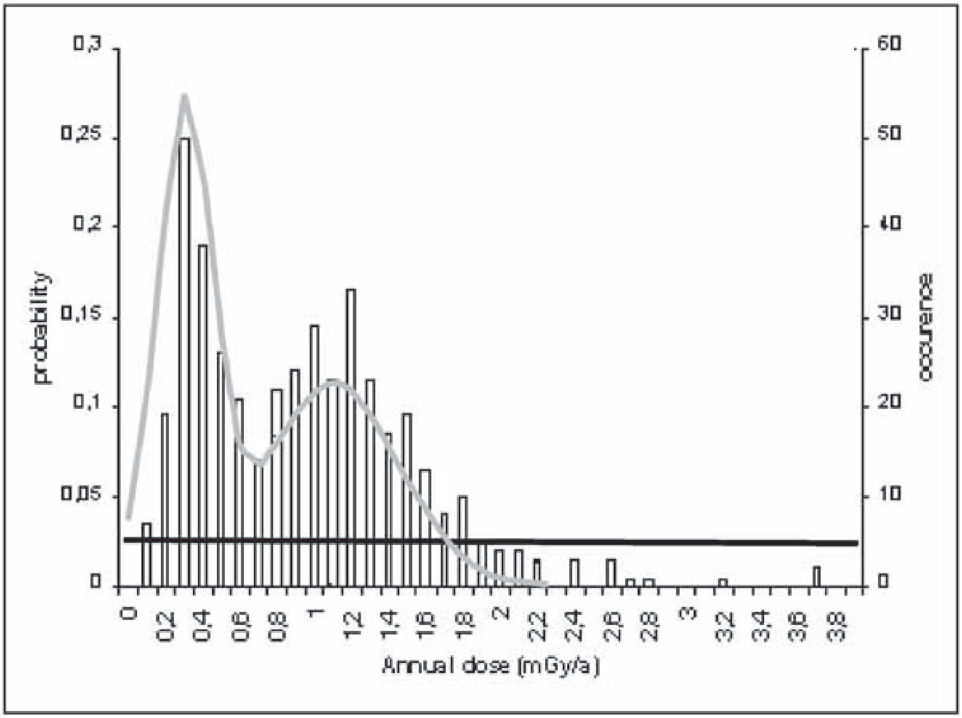

Fig. 1 – Model of gamma external dose. The histogram is based on a compilation of the occurrences collected on a database of 50 references. The grey line shows a uniform distribution of probability between 0 and 4 mGy/a based on the minimum and maximum values of the database. The black line shows a distribution of probability simulated the histogram and based on two gaussians with the respective mean and standard deviation: 0.30 ± 0.15 and 1.10 ± 0.35.

tions centred on the peaks with the following parameters 0.30±0.15 mGy/a and 1.10±0.35 mGy/a, or an uniform distribution between 0 and 4 mGy/a. Our choice was to use an other information, the gamma dose inside the artefact. We can assume that usually the inner dose and the external dose are close. It is relatively exact for the common ceramics where the raw product region and the settlement are in the same geological region, and less for prestige ceramics which are object of long trade exchanges. To minimise such difference, we put a large uncertainty, namely 1.3 mGy/a, corresponding of the variance of the uniform distribution suggested above.

3.3 *Application*

A statuette of a standing young woman of "Tanagra" style, dated from 5th-4th c. BC on the base of stylistic information (L. 24193, DAGER, Louvre

Museum). It means size was evaluated to 10 cm. The corresponding fractional dose for each radioelement K, U, Th was calculated. The internal gamma dose calculated from alpha counting (for uranium and thorium) and X-ray analysis measurements (for potassium) is equal to 2.0 mGy/a. The other irradiations (alpha, beta and cosmic ray) give a dose of 5.3 mGy/a. The equivalent dose received by the statuette from its last heat was 12480 mGy. Using for the external gamma dose a value of 2.0±1.3 mGy/a, the estimated age is of 1710 years with an uncertainty due to the external gamma of 13.5%. As the other uncertainties, as described by Aitken (1985), are around 2 to 5%, the total uncertainty is of 18% or 300 years.

4. Distribution of the estimated age

Mainly, the estimated age is the manufacture heat for a statuette. For a cooking pottery or an element of kiln, it is its last use. In the case of original object, the age is based on the typology, and often given by the curator before the luminescence study. It is modelled by a uniform distribution on the suggested period. It is possible to define an exposition time, corresponding to the duration when the object is not buried. Without other information, fire, inducing a reset of the signal, can occur for one artefact uniformly during the exposition time. Hence, the distribution of corresponding age for the objects of a given typology is similar to the distribution of no buried objects as a function on the time. As a first approximation, the distribution can be taken as a decreasing exponential as suggested by Orton (1980). The ratio of the exponential area on the uniform distribution area must be equal to the assuming proportion of re-heat object. It is may be the most difficult to estimate.

For a forgery, a roughly estimation of the age is a uniform distribution from present to some century is assumed. It is not need to distinguish some re-heat from the presumed manufacture age. But any information on the evolution of the art market can be hopeful to improve the distribution of the probable ages for the forgery.

5. Additional dose

Our previous estimation of the external gamma dose is based on natural radiation sources. But occasionally, the object could be in presence of radioactive sources. As it occurs only during short time, it is difficult to introduce it as equivalent dose rate beside the natural annual dose. It is the cause, it is introduced as additional dose.

An important origin is the x-ray radiography for scientific purpose, used for heritage purpose since their discovery (1895). Other sources could be the airport control or the absorbed cosmic ray during flight. An other origin is the use of radioactive source to falsify luminescence dating. In this case, the received dose is in agreement with the assuming age. The date of entrance in the collections, any information on the history of the object (scientific study, loan to exhibition), as well as a technical knowledge on the possible delivered doses are need to estimate the probability for such radiation exposition. Yet, the information as proportion of object studied under x-ray as the doses delivered by x-ray equipment are under investigation. Hence it is difficult to give a satisfying distribution of the dose. We assume a uniform distribution, limited by a dose maximum equal to the maximum probable age multiply by the estimated annual dose. This distribution is similar as the object is a fake or an original. In the case of forgery, when a falsification by artificial irradiation can be presumed, a uniform distribution centred on the product of the probable age of equivalent original artefact by the total dose distribution (a uniform distribution between 3 and 8 mGy/a) is added. The ratio of the area of the various events (accidental irradiation, malicious irradiation, without additional irradiation) corresponds to the presupposed respective proportions.

6. Hypotheses testing

From above, the presumed equivalent dose distributions are deduced for both falsified and original object. They correspond to our prior for each hypothesis. Mainly, no assumption is made on the probability of authenticity before measurement. Hence, the two hypotheses are equivalent or their total area is identical. Applying the Bayes's theorem, both prior distributions are multiplied by the distribution of the measured equivalent dose. The degree of authenticity after measurement is then the ratio of the areas of the posteriors for the two hypotheses.

7. Conclusion

In this paper, we presented the help bring by a Bayesian approach to the authentication of Museum artefacts using luminescence dating.

A main interest of use a Bayesian approach is the possibility to bring to the curators and other scientific personnel of the museums a quantitative estimation, even roughly, of our results. It is more useful than given a date with good uncertainty, but lot of qualitative warning and remarks. And it can be easy use by non specialist of luminescence dating to improve their own knowledge.

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ABSTRACT

An important field of the luminescence dating is the authenticity of ceramic art objects. The use of "authenticity" instead of "dating" is due to the ignorance on the ambient radiation, and hence the annual dose. The present paper shows a Bayesian approach able to quantify the degree of authenticity. This approach permits to introduce under mathematical models some assumptions (annual dose, fire, artificial irradiation) previously only presented as qualitative warnings in authenticity reports.