1. Introduction

The main goal of archaeological research is to reconstruct social actions in the past from a more or less coherent sub-sample of material remains survived to the present (Clarke 1968; Barceló 2001). Traces of those actions are fossilized in discrete aggregations of artefacts and ecofacts that can be used as an inference about past behaviour. Our purpose is to understand why someone made something, somewhere and somewhen: if an action A took place at some location L, and at some time T, it should be related with the occurrence of observed material evidence around L that was generated at time T. However, the observed material evidence located elsewhere, and at time T-1 and T+1, could explain why A took place in certain specific spatial location L and not in another place (Barceló 2005; Maximiano 2007).

Thus, the proper location in time and in space of archaeological materials is a necessary requisite for any archaeological investigation. Regrettably, since time travel is not possible, and neither is it to observe the archaeological record as it originally was, through a crystal ball, a very biased subset of material culture, produced by the combined action of depositional and post-depositional processes, was recovered by archaeologists. We must be aware that the sediment alone and the material remains buried in it do not directly provide a solution to our questions. They are just data. Indeed, what is recovered is not necessarily an accurate reflection of what originally existed, but only a partial and altered picture.

2. State of the art

The deformation and incompleteness of the archaeological record is not the only issue to be tackled in the attempt at reconstructing the past.

Traditionally, archaeologists compare counts of artefacts, structures and ecofacts from different activity areas in the site, without taking into account some basic assumptions: the material consequences, which identify the activity areas, can be the result of different past actions, carried out in different spatial units and time-spans. We cannot count and compare this evidence as if it belonged to the same past events, if this condition is not verifiable: it is necessary to examine in depth the archaeological context in which the actions took place in the past.
Furthermore, traditionally archaeologists compare counts of artefacts, structures and ecofacts from different layers or occupation floors. These stratigraphic units consist of material results of different past events which took place in different time spans, but in the same space location; they were carried out by inhabitants (not necessarily the same) during their site occupation. Thus, in order to verify if different layers can be comparable, an understanding of their formation and deformation processes, as well as the knowledge of their chronology and material features are needed.

The same issue characterizes both a micro scale (the intra-site dimension) and a macro scale (inter-site dimension): despite archaeologists traditionally compare counts of artefacts, structures and ecofacts from different sites, it is meaningful to take into account some widespread agents of differentiation, among others, the chronology – the different space-time units to which the archaeological record and its depositional and post-depositional processes belong – as well as the environmental processes and landscape features.

Regarding chronological studies, in the last decades archaeologists have started to compare counts of radiocarbon dates from different sites and regions in order to infer demographic trends (Gamble et al. 2005; Turney et al. 2006; Shennan, Edinborough 2007; Buchanan et al. 2008; Smith, Ross 2008; Oinonen, Pesonen, Tallavaara 2010; Peros et al. 2010; Steele 2010; Tallavaara, Pesonen, Oinonen 2010; Johnson, Brook 2011; Williams 2012; Armit, Swindles, Becker 2013; Martínez, Flensborg, Bayala 2013; Shennan et al. 2013; Armit et al. 2014; Barceló, Capuzzo, Bogdanović 2014; Crombé, Robinson 2014; Borrell, Junno, Barcelo 2015). Regrettably, the reliability of such analysis cannot be high, if sample pre-screening rules are not applied. In fact, the archaeological signal can be altered since dates are frequently not filtered in order to guarantee equal representativeness of archaeological contexts in each region.

A deeper understanding of depositional and post-depositional formation processes seems therefore crucial in archaeology in attempt to reconstruct the past society: they provide information about the complex amalgam of human behaviour and natural force that produces the incomplete and distorted reflection of the past. Events are located in space and time (space-time unit), which also define the extension of their material evidence (including the size of the sampled area); such particular “historical” events should be understood in terms of the occurrence of social actions that were performed by someone who produced something somewhen (time) and somewhere (space). This implies that the duration of an historical period can be estimated in terms of the temporal duration of performed social actions.

Consequently, the first step for the reconstruction of what happened is the quantification of those material consequences of past actions, through the keywords of count and frequency.
3. Quantifying the past: count and frequency

The quantitative definition of archaeological evidence implies that archaeologists face two possible occurrences. When these traces belong to the same variable or unit, as spatial location or time interval, their enumeration can be defined as count (Lindsey 1995). Moreover, if the spatio-temporal unit in which the action took place in the past is also known, the intensity of the action can be calculated as frequency. Hence, in the latter case, the evidence can be quantified through a count of frequency: this allows the archaeologists to identify space and time patterns documented in the sediment.

In a macro spatial scale (inter-site dimension) the enumeration of frequency applied to the material artefacts can be used as a tool in attempt to define cultural groups and the boundaries that separate them; on the contrary, in a micro spatial scale (the intra-site dimension), the accumulation of some material items on the ground surface can describe specific functional areas in a settlement and the intensity/repetition of the past actions (Fig. 1).

According to a temporal perspective, the count of frequency can be useful in order to measure events through the detection of discontinuities between those events and the contiguous ones. As Buck and Millard (2004, V) noticed, all the methods share a common factor: «they take a collection of dates or temporal relationships for a series of individual events and combine them with other information to synthesize a chronology which may include the inferred dates of events for which no direct dating evidence is available».

Before the introduction of absolute dating techniques, several attempts were made in order to “quantify” spatio-temporal dynamics in prehistoric
archaeology. Nelson first managed to create chronological types, useful for measuring time (NELSON 1909). Through the typo-chronological seriation, he selected attributes (shape, decoration, colour and design of artefacts) that changed through time and across space. Such variations were used to measure the temporal duration of events that took place at some specific location, like, for instance, the adoption of some pottery decorations (Fig. 2).

With the introduction of radiocarbon dating at the end of the 1940s, the material evidence of social actions started to be measured in a quantita-
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tive way. The first attempts at quantifying the duration of events summing a group of radiocarbon estimates were introduced by Ottaway (Ottaway 1973; Aitchison, Ottaway, Al-Ruzaiza 1991). The author introduced the concept of culture *floruit*, which is the period of time when the 50% of artefacts characterizing a specific group of people from a specific geographical area (“a culture”) were produced (Fig. 3). This can be represented using a frequency distribution of the number of characteristic artefacts per time unit. The *floruit* of an archaeological site can be defined in exactly the same manner (Aitchison, Ottaway, Al-Ruzaiza 1991).

4. Archaeological observables as events: the case study of accumulation process

The material evidence which composes the archaeological record is formed by the intersection of social agents, actions and natural processes in space and through time. Therefore, the notion of event or success should be introduced (Barceló 1991, 1993; Andresen, Madsen, Scollar 1993; Doerr et al. 2003; Mantegari 2010). Events are not observable; they are latent and observed through, but not defined by, noisy data. An «event is thus a theoretical construct» (Parnell et al. 2008, 1873) materialized into the archaeological sediment. An assemblage of bones, potsherds or lithic débitage can be identified as archaeological events: it was stored due to behavioural processes, natural forces or post-depositional changes (as trampling, scavenging or erosion) and their material consequences are an example of accumulation. Some particular actions were repeated numerous times in the past, and each repetition produced evidence, which is partially detectable. It follows that accumulation can be caused by both natural and human factors. In the second case, we should refer to intentional and unintentional accumulation.

The intentional accumulation process involves a spatial pattern associated with storing behaviour; when we take the decision to store things at a specific place, the material effects of that action are distributed according to a regular spatial distribution and a regular time span, which can be measured by radiocarbon dates. Furthermore, this material evidence is denser near the place where the action was supposedly performed and less dense or even dispersed far from the central place (Tobler 1970). However, the material consequences of the deliberate human decisions about artefact placement suffer the effects of various kinds of post-depositional and taphonomic processes involved in the formation of archaeological record. They can influence also the spatial pattern and extension/distribution of an unintentional accumulation, such as a combination of random activities carried out during the past, according to different reasons. In the most likely case, everything may have occurred
In conclusion, as previously said, during the interpretative process archaeologists need to take into account that their reconstruction is affected by uncertainty at any stage. A strong combination of post-depositional and taphonomic processes can alter the sediment and modify its preservation status (Fig. 4). Thus, we cannot know a priori if the material consequences, which now compose the archaeological record, completely reflect the sequence of the past social events unfolded over the time. Consequently, these interpretative processes are called stochastic and the scenario which archaeologists can reconstruct has to be probabilistic.

5. Modelling the accumulation processes

The archaeological observation represents the material consequence, preserved in the present, of a single repeated event (accumulation) which took place at the site during a particular time span. Their quantity can be considered as an estimation of the number of repetitions of the process in the past.
Throughout the frequency of material results per unit of space and time, we should be able to estimate the probability with which the action took place and its importance and intentionality among other actions. For instance, the occurrence of a broken shred of an animal bone of species s, with shape x at a location z and the most abundant presence of a lithic tool with texture r at a neighbour location are events: indeed, some social actions were performed at this spatial and temporal location (event), with the purpose of producing a subsistence good, maintaining some conditional properties of that place, or with the intention of materially fixing some reproductive behaviour (BARCELÓ 2005; BARCELÓ, MAXIMIANO 2013). Two possible scenarios can occur throughout time: the material consequences observable can be the results of successive repetitions of the same past action in different locations or they can be produced by successive repetitions of different past actions.

In the first case, we can assume that this evidence was distributed as a Poisson process. According to a statistical definition it is a stochastic process which counts in a probabilistic way the number of events and the times over which these events occur in a given time interval. It is expressed through the following formula:

\[ f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \]

where the random variable k is the number of occurrences of the event in an interval; the probability of the event occurring x times (x successes) over an interval is given by f and \( \lambda \) represents the mean number of successes over the interval.

On the other hand, when the material consequences observable are the results of successive repetitions of different past actions in different locations, archaeologists traditionally consider that this evidence is distributed according to a multivariate (bivariate) normal distribution, as proposed by BARCELÓ, MAXIMIANO 2007; 2008); this is a generalization of the one-dimensional (univariate) normal distribution to two dimensions.

A pair of random variables x and y have a bivariate normal distribution if their joint probability density is given by the following formula:

\[ f(x, y) = \frac{1}{2\pi \sigma_1 \sigma_2 \sqrt{1 - \rho^2}} \left( \frac{x - \mu_1}{\sigma_1} \right)^2 - 2\rho \frac{(x - \mu_1)(y - \mu_2)}{\sigma_1 \sigma_2} + \left( \frac{y - \mu_2}{\sigma_2} \right)^2 \]

where the joint probability of those events (x and y random variable) occurring x times (x successes) over an interval is given by f.
In the end, archaeologists traditionally consider the appropriateness of these two models in certain above-mentioned conditions a universally valid assumption. Since the correspondence between the spatial pattern of an archaeological data set (in the case study the material evidence of past actions) and a statistical distribution or process should not be considered as an *a priori* condition, we highlight the importance of fitting the models.

Therefore, the applicability of this model is validated through respectively the Kolmogorov-Smirnov test (KOLMOGOROV 1933; SMIRNOV 1948) to fit the Poisson processes and the Mardia’s test (MARDIA 1970; 1985) in order to fit the bivariate distribution. The first one is a goodness-of-fit test that verifies whether a given distribution is not significantly different from a hypothesized one. Through the comparison between the statistical distribution/process showed by different datasets, it is possible to verify if the statistical hypothesis fits the analyzed sample population. In the case of Mardia’s test, through the analysis of skewness and kurtosis, it is possible to test if the bivariate normal distribution fits the observed datasets. In these data, kurtosis decreases proportionally to the increase in standard deviation of the distribution. That means that as soon as spatial entropy increases, concentration decreases, without affecting the regular modality of that distribution.

If these models fitted our data archaeologists can consider the material evidences preserved in the archaeological record as a result of an intentional process of accumulation. On the contrary, when the models have not fitted our data, the material evidence is produced by an unintentional accumulation process.

The unintentional accumulation may be the result of intentional actions carried out in the past according to randomness. Nevertheless, it would be more logic to think of social actions that were performed in the past unintentionally and that resulted in the random distribution of archaeological evidence. If the archaeological record is the result of an intentional choice to obtain a system based on the randomness, we will deal with a complex system which presents, according to MITCHELL (2009, 13) «large networks of components with no central control and simple rules of operation giving rise to complex collective behaviour, sophisticated information processing and adaptation via learning or evolution». Otherwise, this unintentional accumulation could be produced by the effect of natural phenomena (taphonomic processes) that can alter the original spatial intentional distribution of artefacts and ecofacts (cfr. *infra* § 7).

On the contrary, as previously mentioned, when an intentional process of accumulation took place during the past, two possible occurrences may be inferred: either the existence of a homogeneous or inhomogeneous process of accumulation.
6. THE INTENTIONAL ARCHAEOLOGICAL ACCUMULATION: HOMOGENEOUS AND INHOMOGENEOUS PROCESS

In all the historical events, it is assumed that each action was repeated many times over a certain time span; however, the specific probability that an action left some material evidence is expected to be very low, for reasons specific to the action. Indeed, detecting an individual action is a hard work due to the perishable evidence that it generates. Therefore, we need to focus basically on collective actions whose consequences can be detected in the archaeological record.

Nonetheless, if the expected number of repetitions of that action in the past was similar to the most probable frequency predicted by the theoretical model based on Poisson distribution, observed frequencies of archaeological items are seen as realizations of a single homogeneous process. In this case, we assume to know the frequency of material evidence at a given space-time unit, and the probability of occurrence of such materiality in this place and moment is proportional to the duration of the event and the spatial extension of the sample area. Furthermore, the probability that the action occurred is independent of any other repetition of the same action. Finally, under these circumstances the average frequency of occurrence of the event ($\lambda$) should be constant, regardless the occurrence of events up to the time of observation. If these conditions are verified and a homogeneous process took place in the past, we can reach, only in this case and in a probabilistic way, a deeper knowledge of the community which inhabited our archaeological context.

As firstly highlighted by the Nelson’s and Cook’s work (published for the first time in Schiffer 1975, 840) the amount of discarded material (in particular the tableware pottery) can be seen as a function of the length of site occupation, the size of the group that inhabited the site and the rate at which specific artefacts were discarded (Varien, Ortman 2005, 132-133). Furthermore, in the 1970s, Schiffer formalized the study of these interactions with a formula known as the “discard equation” or “Cook’s law”:

$$T_D = \frac{St}{L}$$

where $T_D$ is the total discard of an artefact type; $S$ is the systemic number, or the number of artefacts of a given type in use at any given point in time; $t$ is the length of time over which the discard takes place; and $L$ is the life-use of the artefact type. Moreover, a key role was played by the so-called “Clarke Effect”: it describes the statistical tendency for the variety of discarded artefacts to increase directly with a settlement’s occupation span (Clarke 1968).

Consequently, although interesting, the suggestion provided by the so-called discard equation has to be proved. It is practical only under some
conditions: it requires the complete site excavation or at least the excavation of a large continuous area, because from a single dig one cannot expect to recover many sherds from the same vessel. A different scenario arises when the material consequences that characterize the archaeological record are the result of an inhomogeneous process. In this case, the frequency of the action in a particular interval is proportional to its duration (time) and the probability that the action occurred is independent of any other repetition of the same action. In the end, differently from the homogeneous process, the average frequency of occurrence of the event ($\lambda$) should not be constant, regardless of the occurrence of events up to the time of observation.

Explained method can be applied to a wide range of case studies dealing with formation and deformation processes involving the archaeological record. For instance, in a settlement the occurrence of different markers, such as ceramic types, bones, lithic industry and household structures, could be linked to several specialized activities carried out in specific functional areas. In the light of such evidence, through a statistical analysis of items frequency, we may be able to identify if their spatial distribution is the result of a homogeneous or a non-homogeneous process. Through the frequency of our archaeological evidence we can also predict how many times that social action was repeated in the past. For instance, some action of production or consumption that was repeated in the past numerous times. At each repetition certain material evidence specific to that action was materialized. We refer here to work events (hunting, manufacturing of ceramics, jewellery or weapons, building a house, etc.), in which goods or refuse material was generated as a result of human activity. In order to reach the quantification goal, we have to be aware of the spatial and temporal defined unit. However, not all the archaeological issues relate to a count of frequency. In some cases two possible outcomes (presence/absence of an archaeological marker) are enough to reconstruct the probability of that phenomenon under study occurred during the past. This is the case of the adoption of innovation.

7. Archaeological observables as events: the case study of expansive phenomena

It is meaningful to highlight that the temporal variable need to be also taken into account for a better quantification of spatio-temporal distributions of archaeological data. Since social actions are characterized by a specific location in space and in time, it is relevant to also approach the analysis of the temporality of such actions in a quantitative way. In order to calculate the specific time-span when a certain event took place, it is essential to adopt a scientific and objective approach. In this field radiocarbon measurements can provide a reliable tool to locate specific events in time, such as the intro-
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![Figure 5](image_url)

**Fig. 5 – Effects of post-depositional processes on archaeological evidences effects (Brantingham, Surovell, Waguespack 2007).**

duction of a new artefacts and episodes of cultural change among prehistoric societies. Specifically, we focus on the case study of dispersal phenomena which correspond to the diffusion of people, objects and ideas across space and over time.

When a system expands through time, a certain degree of dependence between locations can be foreseen, and this dependence is exactly what gives unity to the process (Barceló, Capuzzo, Bogdanović 2014). Such dependence is a consequence of the previously mentioned Tobler’s law according to which “everything is related to everything else, but near things are more related than distant things” (Tobler 1970). Here the idea of “distance” is to be considered in both temporal and spatial terms. This principle constitutes a key-concept in order to explain the spatio-temporal dynamics of any series of events in the archaeological record. We refer to expansive phenomena as dynamical systems in which every location at some well specified underlying space has a distinctive behaviour through time. Our definition comes from the mathematical concept of expansivity, which formalizes the idea of points moving away from one-another under the action of an iterated function.

Thanks to the geo-statistical analysis of radiocarbon datasets associated to presence of archaeological markers it has been possible to reconstruct a wide series of expansive phenomena; among them, a great effort has been dedicated in the last decades to the study of one of the most relevant dispersal process in human History, i.e. the diffusion of agriculture and the process of neolithization (Ammerman, Cavalli-Sforza 1971, 1984; Gkiasta et al.
2003; Russell 2004; Pinhasi, Fort, Ammerman 2005; Dolukhanov et al. 2005; Bocquet-Appel et al. 2009; Isern et al. 2014). In recent years the study of expansive phenomena in archaeology has also included episodes of cultural change, for instance, the introduction of cremation burials among European Bronze Age societies (Barceló, Capuzzo, Bogdanović 2014) or the diffusion of specific material cultural markers, such as early pottery technology in Eurasia (Silva et al. 2014) and a Bronze Age pottery typology in Western Europe (Capuzzo 2014). The starting point of these studies is the analysis of the first occurrence of specific archaeological features (first traces of agriculture, 2nd millennium BC cremations, new pottery typologies, etc.) in a stratigraphic layer closely associated to one or more 14C measurements.

It follows that, to ensure the reliability of these analyses we also need to take into account the bias of deformation processes, which still plays a key role in the accumulation process and in its preservation. Its effects should be recognized and modelled according to our knowledge on the most likely past scenario.

8. Modelling the deformation processes

Among others (Mameli, Barceló, Estevez 2002; Brantingham, Surovell, Waguespack 2007: Fig. 5), G. Carver (2004, 2006a, 2006b, 2015) studied the effects of taphonomic transformations through statistical tools; in particular, he modelled the post-depositional loss of artefacts and their movement through the application of “Markov chain”. The requirements of his model are two discrete dependent events (the initial and transformed states, in particular the deposit and recovery sample) and a probability of transformation from one state to another, linked for example to the environmental changes (soil acidity, rising groundwater, particularly deep frost, infestation as a result of change in climate or land-use, etc.). If these phenomena cannot be identified specifically, their cumulative effects can be modelled randomly.

Once these environmental factors have been identified, probability values of decay must be estimated for a given time interval (year, decade, century). A central condition is that events are not independent: indeed, the recovery sample is related to the deposited one and it is not possible for the retrieved population to exceed the deposited population. Markov chain, named after Andrey Markov, is a mathematical system that undergoes transitions from one state to another on a state space. It is a random process usually characterized as memory-less: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of “memory-less-ness” is called the Markov property.

Finally, if the archaeological counting of the material evidence is expressed through frequencies for time-space unit, and the temporal discontinuities occur in a spatial distribution of accumulation, the homogenous process can be studied
The quantification of spatio-temporal distributions of archaeological data through the statistical tools analysed. The implementation of this model could represent a useful keystone in attempt to reconstruct the most likely scenario of the past, as it takes into account and reconstructs the depositional and post-depositional processes which formed and altered that archaeological record.

9. Conclusion

Material evidence of social events that took place during the past experienced an amalgam of natural and cultural processes which involve alterations in the intrinsic characteristics of the deposit. The absence of a time machine prevents us to know how an archaeological record was at the exact time of its formation; thus, we need to reconstruct the set of actions and interactions among the different depositional and post-depositional agents. Nevertheless, the simple enumeration of the number of observed evidence in the present hardly gives us any information regarding the way the action was performed in the past. The action may have been performed once, and all remains are just material consequences of it, or the action may have been performed repeatedly many times, and all observed remains may be a kind of palimpsest. The inference about the historical relevance of that action can be addressed only if we translate observed counts of different artefacts into the historical frequency of that action: an expression of the number of times a certain event took place during a defined time-span. These ideas are not very usual in archaeological research. Counts are used as direct evidence of the historical relevance of actions, without taking into consideration neither the probabilities of repetitiveness or synchronicity of different actions nor the duration of the processes.

This paper is aimed at highlighting the relevance of statistical tools for the reconstruction of the past, trying to “see” what cannot be “seen”. Specifically, our goal was to understand whether the archaeological record we observe in the present is the result of a homogeneous process that happened in the past. If this were the case, observed data (counts) should fit theoretical distributions and only then we would assume the Markov condition, that would allow us to consider the number of observed evidence in terms of the time needed to generate that accumulation (frequencies). Then, the amount of observed data can be statistically used to infer the nature of the historical event. In the light of the results of such hypothesis testing, we have proposed a theoretical and methodological approach for the study of accumulation processes and expansive phenomena throughout the presence of archaeological markers in the archaeological record.

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Acknowledgements

This research has been possible thanks to the funding of the Departament d'Universitats, Recerca i Societat de la Informació of the Generalitat de Catalunya. It is also part of the research group AGREST (2014 SGR 1169) and the project “Social and environmental transitions: Simulating the Past to understand human behavior” funded by the Spanish Ministry for Science and Innovation, under the program CONSOLIDER-INGENIO 2010, CSD2010-00034. We also acknowledge funding from the Spanish Ministry of Science and Innovation, through the Grant No. HAR2012-31036 awarded by Prof. Juan Antonio Barceló.

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ABSTRACT

Traces of past social actions, detectable in the archaeological record, are the material evidence through which we can infer social and economic patterns of ancient societies. These categories can be investigated in both time and space using a probabilistic statistical approach. In an attempt to quantify the results of archaeological processes we distinguish the terms of count and frequency, which is not common in archaeology, focusing particularly on the latter. In this framework we are able to calculate the number of times a certain event took place in relation to the length of the time interval during which the event is repeated. In addition, the statistical tools allow us to understand if the observable material evidence is the result of a particular archaeological phenomenon (accumulation) that can fit a statistical distribution or process (Poisson process and multivariate normal distribution).